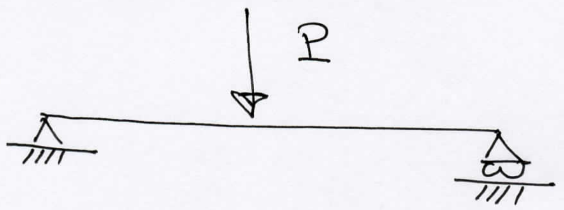
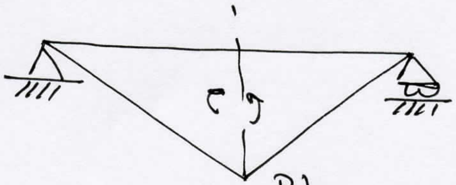
Mecanismos de colapso.

Hipótesis simplificativas:

1.- El diagrama de momento-curvatura será bilineal con la rama elástica recta hasta Mp y la rama elastoplástica horizontal.

2.- Despreciaremos la influencia de los efectos del axil y del cortante frente al efecto de flexión.

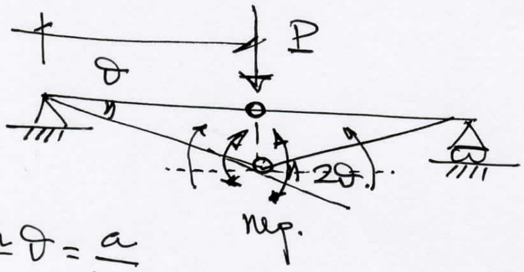




$\frac{P \cdot L}{4}$

$$M_p = \frac{P \cdot L}{4} \longrightarrow \boxed{P = \frac{4M_p}{L}}$$

Principio trabajos virtuales:



$tg\,\vartheta \simeq \vartheta = \frac{a}{L/2}$

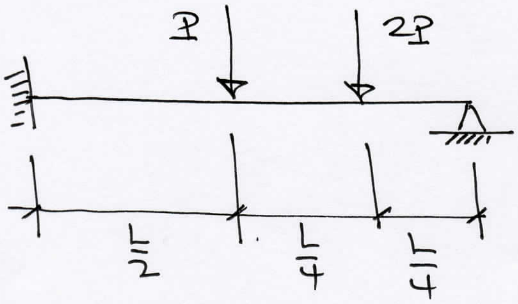
$W_1 = P.a = P.\vartheta.\frac{L}{2}$

$W_2 = -M_p.2\vartheta$

$$\left(P.\vartheta.\frac{L}{2}\right) - M_p.2\vartheta = 0$$

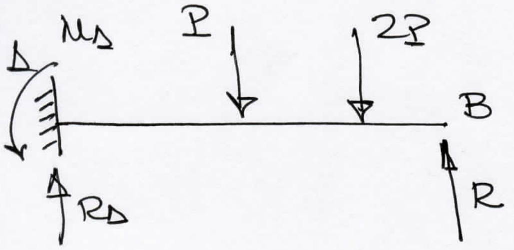
$$\boxed{P = \frac{4M_p}{L}}$$

$$\boxed{P.a = M_p.2\vartheta.}$$



Para que la estructura se agote sean necesarias tantas rótulas como reacciones hiperestáticas más una.

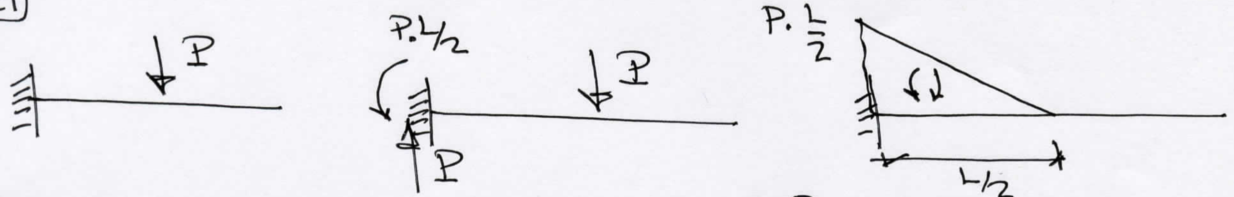
$$GH = 3 - 2 = 1.$$



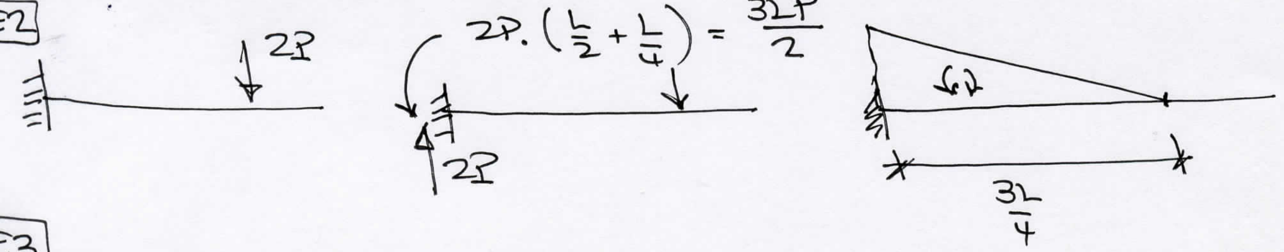
$$\overset{\downarrow}{v}_B = 0$$

$$\overset{\downarrow}{v}_B = \cancel{v_A}^{0} + \cancel{\theta_A}^{0} \cdot d + \int_A^B \overline{X} \cdot d(s) \cdot ds.$$

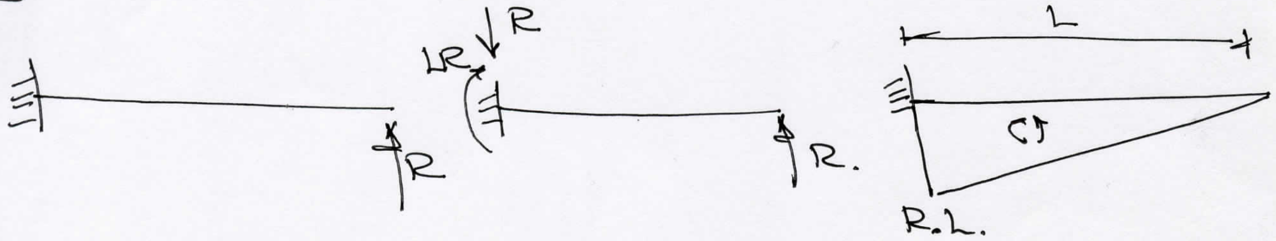
E1



E2



E3

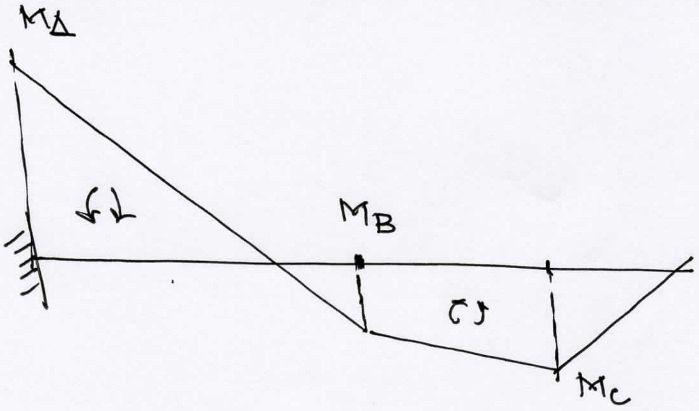


$$0 = \frac{1}{EI} \left[\frac{1}{2} P \cdot \frac{L}{2} \cdot \left(\frac{2}{3} \frac{L}{2} + \frac{L}{2} \right) + \frac{1}{2} \frac{3L}{4} \frac{3LP}{2} \left(\frac{L}{2} + \frac{2}{3} \frac{3L}{4} \right) - \frac{1}{2} R.L. \frac{2}{3} L \right].$$

$$\boxed{R = \frac{101}{64} P}$$

$$\Sigma F_v = 0 \qquad R_A + R = P + 2P$$

$$\boxed{R_A = \frac{91}{64} P}$$

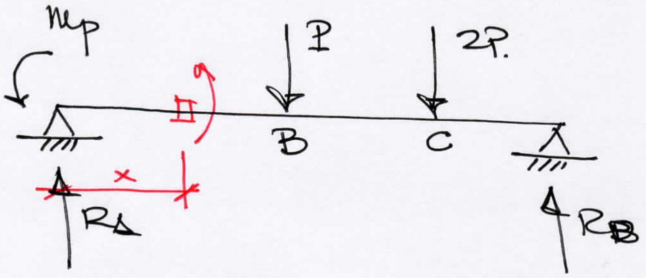


$$M_A = \frac{27}{64} P.L = 0'42 P.L.$$

$$M_B = \frac{37}{128} P.L = 0'28 P.L$$

$$M_C = \frac{101}{256} P.L. = 0'39 P.L.$$

La primera rótula se forma en el punto donde el momento flector es máximo.



ISOSTÁTICA.

TRAMO AB.

$\Sigma \bar{M} = \bar{0}$ $\quad \overset{\curvearrowright}{M}(x) + M_p - R_A x = 0$

$\overset{\curvearrowright}{M}(x) = R_A . x - M_p$ *

en B $\; x = \frac{L}{2}$

$\overset{\curvearrowright}{M}(x) = R_A . \frac{L}{2} - M_p =$

$= \left(P + \frac{M_p}{L}\right) . \frac{L}{2} - M_p =$

$= P.\frac{L}{2} + \frac{M_p}{2} - M_p =$

$= \left(P.\frac{L}{2} - \frac{M_p}{2}\right)$

$\Sigma \bar{F}_V = \bar{0} \qquad R_A + R_B = 3P \quad (1)$

$\Sigma \bar{M}_A = \bar{0} \qquad M_p + R_B . L - P.\frac{L}{2} - 2P.\frac{3L}{4} = 0 \quad (2)$

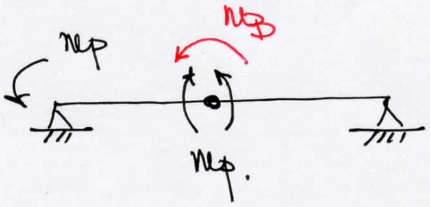
$$\boxed{R_A = 3P - \left[2P - \frac{M_p}{L}\right] = P + \frac{M_p}{L}}$$

$$R_D = \left[2P - \frac{M_p}{L}\right]$$

$$\overset{\curvearrowleft}{M_B} = M_p - R_A . \frac{L}{2} = M_p - \left(2P - \frac{M_p}{L}\right) . \frac{L}{2} = \left(\frac{M_p}{2} - \frac{P.L}{2}\right)$$

$$\overset{\curvearrowright}{M_C} = R_D . \frac{L}{4} = \left(2P - \frac{M_p}{L}\right) . \frac{L}{4} = \left(\frac{PL}{2} - \frac{M_p}{4}\right)$$

Supongamos que colapsa en B:



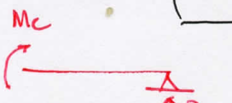
$\boxed{M_p = -M_B}$

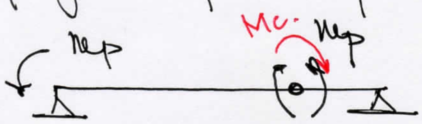
$$M_p = \left(-\frac{M_p}{2} + \frac{P.L}{2}\right)$$

$$\longrightarrow \quad 2M_p = -M_p + P.L$$

$$\boxed{P = \frac{3M_p}{L}}$$

Supongamos que colapsa en C:





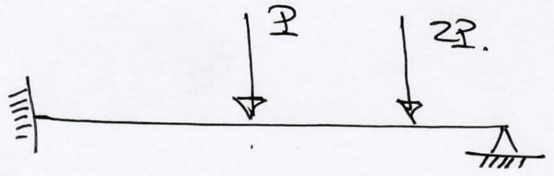
$\boxed{M_p = M_C}$

$$M_p = \frac{P.L}{2} - \frac{M_p}{4}$$

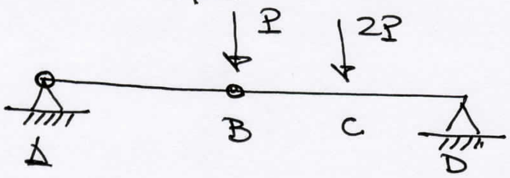
$$\boxed{P = \frac{5}{2}\frac{M_p}{L}}$$

Formación de mecanismos.

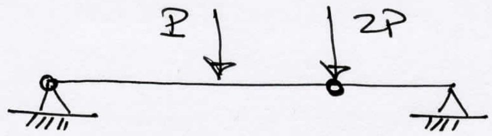
GH = 1. nº RÓTULAS = 2



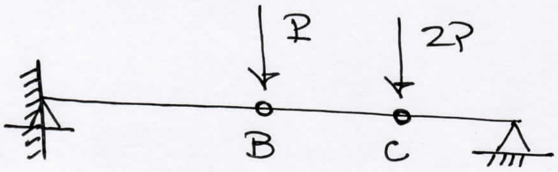
(I) EN A y EN B.



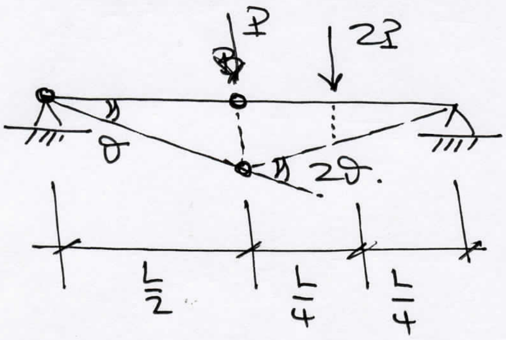
(II) EN A y EN C.



(III) EN B y EN C.



(I)



$$(Pa + 2P.b) = M_p(\theta + 2\theta)$$

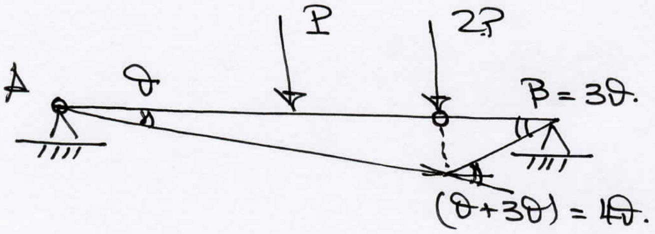
$$\operatorname{tg}\theta = \frac{a}{L/2} = \frac{2a}{L} \longrightarrow \theta = \frac{2a}{L} \quad \boxed{a = \frac{L}{2}\theta.}$$

$$\operatorname{tg}\theta = \frac{b}{L/4} = \frac{4b}{L} \longrightarrow \theta = \frac{4b}{L} \quad \boxed{b = \frac{L}{4}\theta.}$$

$$\left(P.\frac{L}{2}\theta + 2.P.\frac{L}{4}.\theta\right) = M_p(\theta + 2\theta)$$

$$\boxed{P = \frac{3}{L}M_p}$$

II EN A y C.



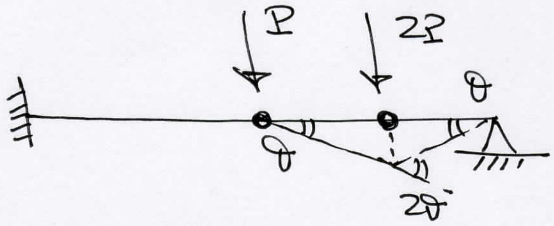
$\operatorname{tg}\theta = \dfrac{a}{L/2} = \dfrac{b}{3L/4}$ $\quad b = \dfrac{3L}{4}\cdot\theta.$ $\quad \operatorname{tg}\beta = \dfrac{b}{L/4} = \dfrac{3L/4\cdot\theta}{L/4} = 3\theta.$

$$(P\cdot a + 2P\cdot b) = M_p\cdot(\theta + 4\theta)$$

$$\left(P\cdot \frac{L}{2}\theta + 2P\cdot\theta\cdot\frac{3L}{4}\right) = M_p\,(\theta + 4\theta)$$

$$\boxed{P = \frac{5}{2}\frac{M_p}{L}}$$

III EN B y C.



$\operatorname{tg}\theta = \dfrac{a}{L/4}$ $\quad a = \theta\cdot\dfrac{L}{4}$

$$2P\cdot a = M_p\cdot 2\theta$$

$$2P\cdot\theta\cdot\frac{L}{4} = M_p\cdot 2\theta.$$

$$\boxed{P = \frac{6M_p}{L}}$$

La solución será la que requiera una carga menor → EL II

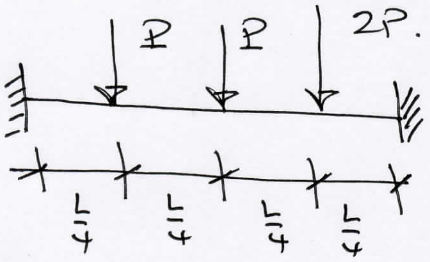
$$\boxed{P = \frac{5}{2}\frac{M_p}{L}}$$

Una viga de 16 m de luz de sección constante de ancho "a" y canto "2a" está sometida a las cargas que se indican.

Se pide:
1) Dimensionar la viga según la teoría plástica.
2) Mecanismos de colapso.
3) Reacciones y momentos flectores en el instante de colapso.

$P = 160\ kN$

$\sigma_f = 10^5\ kN/m^2$

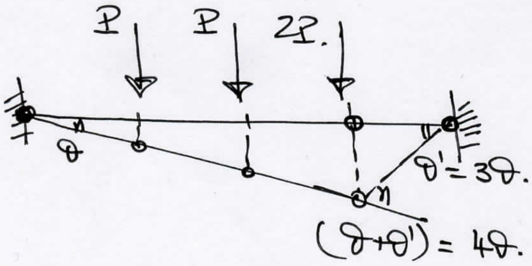


$GH = 4 - 2 = 2$

n^o rótulas $= 3$

n^o de mecanismos posibles: $\binom{5}{3} = 10$.

De los 10 mecanismos posibles hemos de seleccionar aquellos en los que las rótulas que pongamos hagan trabajar mucho a las fuerzas exteriores.



$\tan\theta = \frac{a}{\frac{3}{4}L}$ $\quad a = \frac{3}{4}\cdot L \cdot \theta$

$\tan\theta' = \frac{a}{L/4}$ $\quad a = \frac{1}{4} L \cdot \theta'$

$\frac{a}{\frac{3}{4}L} = \frac{b}{L/2} = \frac{c}{\frac{L}{4}} = \tan\theta$

$\frac{3}{4}\cdot L \cdot \theta = \frac{1}{4} L \theta'$

$\boxed{\theta' = 3\theta}$

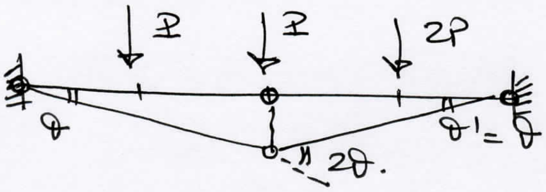
$b = \frac{L}{2}\theta$

$c = \frac{L}{4}\theta$

$$\left(P\cdot\frac{L}{4}\theta + P\cdot\frac{L}{2}\theta + 2P\,\frac{3}{4}L\cdot\theta\right) = M_p\cdot\theta + M_p\theta\cdot 4\theta + M_p\,3\theta$$

$$P\left[\frac{L}{4} + \frac{L}{2} + \frac{3L}{2}\right] = 8M_p$$

$$\boxed{P = \frac{8M_p}{9L/4} = \frac{32M_p}{9L}}$$



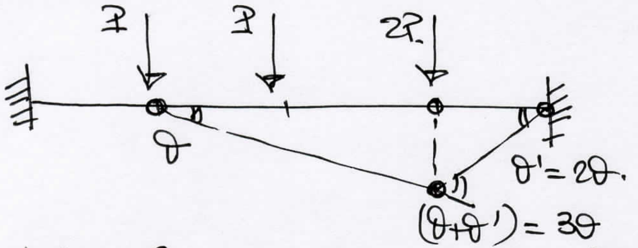
$\theta = \theta'$

$tg\,\theta = \frac{a}{L/2} = \frac{b}{L/4}$ $\qquad a = \frac{L}{2}\theta$

$b = \frac{L}{4}\theta.$

$$\left(P \cdot \frac{L}{4}\theta + P \cdot \frac{L}{2}\theta + 2P \cdot \frac{L}{4}\theta\right) = M_p \cdot \theta + M_p \cdot 2\theta + M_p \cdot \theta .$$

$$P\left(\frac{L}{4} + \frac{L}{2} + \frac{L}{2}\right) = 4M_p \qquad \boxed{P = \frac{4M_p}{\frac{5}{4}L} = \frac{16M_p}{5L}.}$$



$tg\,\theta = \frac{a}{L/2}$ $\qquad \frac{L}{2}\theta = \frac{L}{4}\theta'$ $\qquad \boxed{\theta' = 2\theta.}$

$tg\,\theta' = \frac{a}{L/4}$

$tg\,\theta = \frac{a}{L/2} = \frac{b}{L/4}$

$$P\,\frac{L}{4}\theta + 2P \cdot \frac{L}{2}\theta = M_p \cdot \theta + M_p\,3\theta + M_p\,2\theta .$$

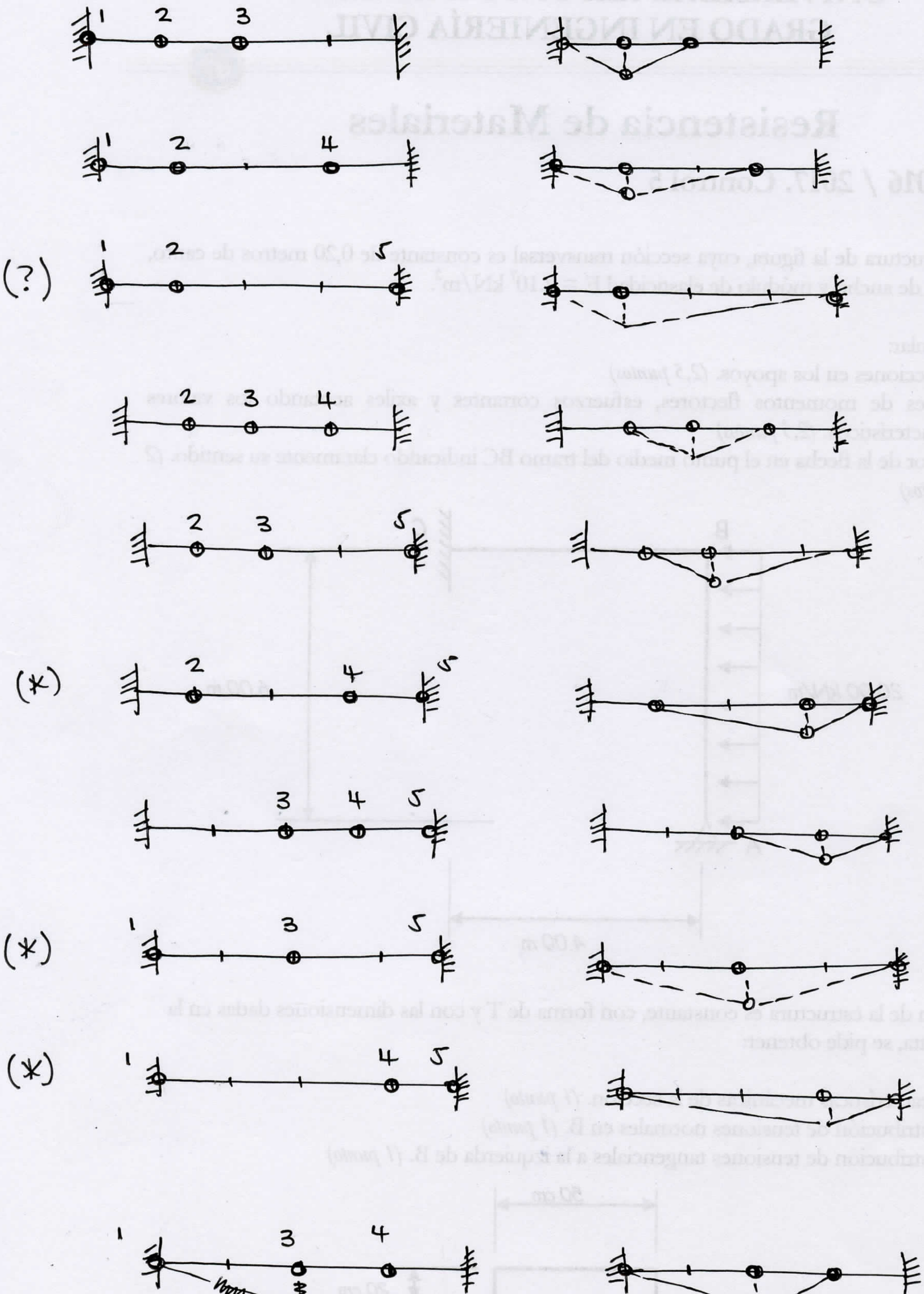
$$P\left(\frac{L}{4} + 2 \cdot \frac{L}{2}\right) = 6M_p .$$

$$\boxed{P = \frac{6M_p}{\frac{5}{4}L} = \frac{24M_p}{5L}.}$$

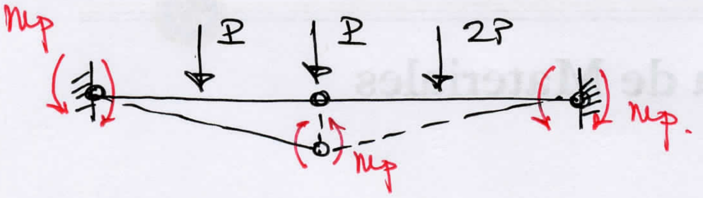
El mecanismo de colapso será el siguiente:

$$\boxed{P = \frac{16M_p}{5L}.}$$

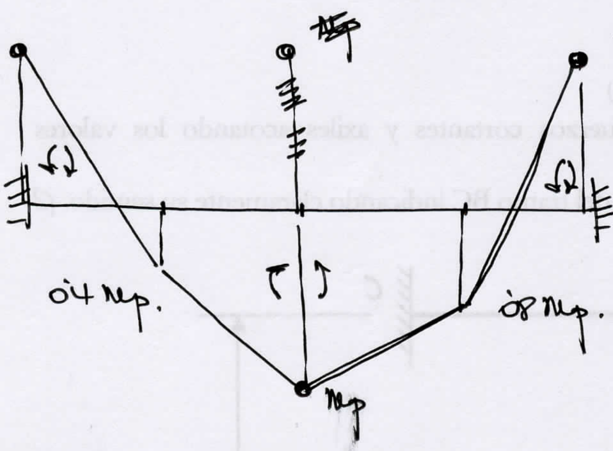
Posibles mecanismos de rotura.



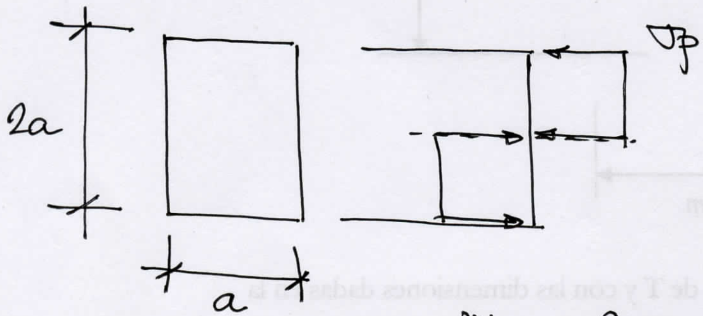
Mecanismo de colapso:



Ley de flectores del mecanismo de colapso:



Momento plástico. (flexión pura)



$$M_p = 2\sigma_p \cdot a \cdot a \, \frac{a}{2} = a^3 \cdot \sigma_p$$

Carga de colapso: $\boxed{P = \frac{16}{5}\frac{M_p}{L}} \longrightarrow M_p = \frac{5}{16} P \cdot L.$

$$a^3 \cdot \sigma_p = \frac{5}{16} P \cdot L$$

$P = 160$ kN.

$\sigma_p = 10^5$ kN.m.

$L = 16$ m.

$$a^3 \cdot 10^5 = \frac{5}{16} \cdot 160 \cdot 16$$

$$a = \sqrt[3]{\frac{5 \cdot 160 \cdot 16}{16 \cdot 10^5}} = 0'2 \text{ m} = \boxed{20 \text{ cm}}$$

VIGAS CONTINUAS.

Consideraciones:

1) Se acepta que la plastificación de un tramo es suficiente para considerar plastificada totalmente la estructura, por lo que la viga continua se estudia tramo a tramo sustituyendo el resto de la viga por un empotramiento.

2) El nº de rótulas será independiente del número de tramos de la viga.

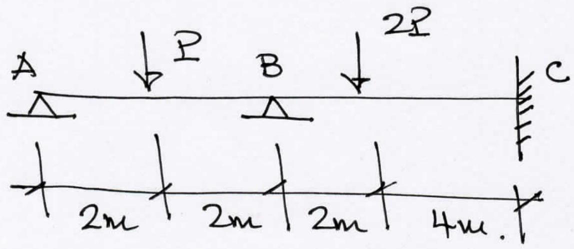
3) Se desprecia ~~la~~ el efecto del cortante ya que se supone que la plastificación ~~[illegible]~~ se alcanza sólo por efecto de flexión.

4) Si la rigidez fuese distinta en cada tramo y se formasen rótulas plásticas en las zonas de unión de tramos contiguos, éstas aparecerían en los tramos de menor rigidez.

4201 a 4250.
4351 a 4375 (4400)

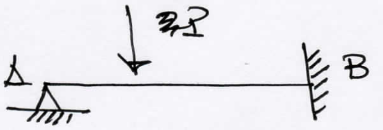
Dada la viga continua de la figura, de sección rectangular 30 x 50 cm sometida a las cargas que se indican, se pide hallar:

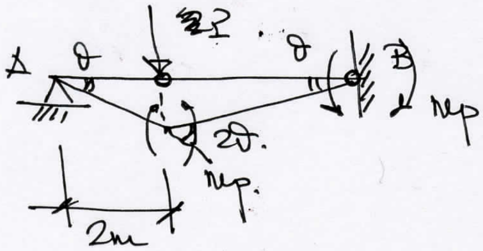
1) La carga de plastificación
2) Dibujar el mecanismo de rotura
3) Ley de momentos flectores en el momento de rotura sabiendo que la tensión de fluencia del material es de 15 MPa.



TRAMO 1. AB

n^o rótulas $= 2.$ (GH $= 1$)





$tg\,\theta \simeq \theta = \frac{a}{2} \qquad a = 2\theta.$

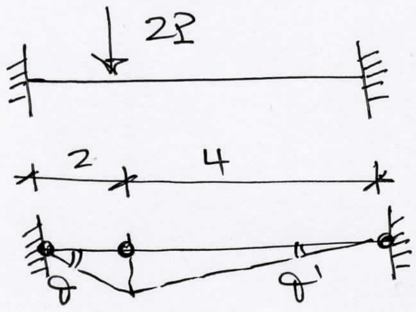
$(P.a) = 2\theta . M_p + \theta . M_p$

$$\boxed{P = \frac{3M_p}{2}}$$

(Trabajo f. exteriores) = (Trabajo rótulas plásticas.)

TRAMO 2 BC.

GH $= 2.$ ⟶ n^o RÓTULAS $= \underline{\underline{3}}$

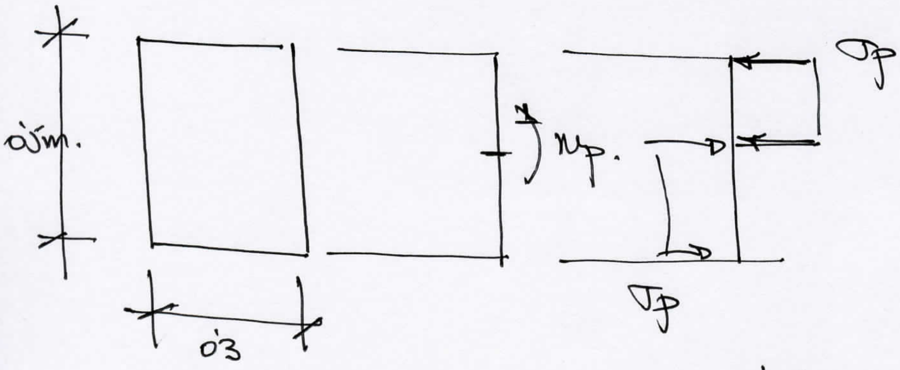


$tg\,\theta = \frac{a}{2}$; $tg\,\theta' = \frac{a}{4}$ $\quad 2.\theta = 4.\theta' \quad \theta' = \frac{\theta}{2}$

$\theta + \theta' = \theta + \frac{\theta}{2} = \frac{3}{2}\theta.$

$2P.a = M_p . \theta + M_p . \frac{3}{2}\theta + M_p . \frac{\theta}{2} \qquad (a = 2\theta) \qquad \boxed{P = \frac{3M_p}{4}}$

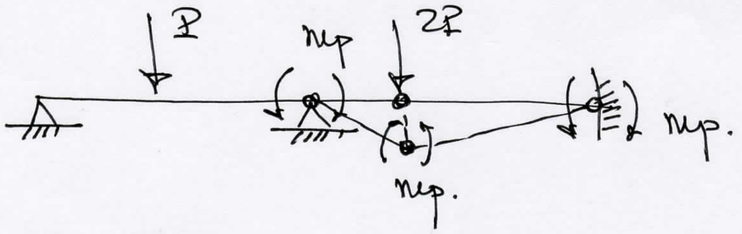
Carga de plastificación. $P = \frac{3}{4} M_p$. (la más pequeña).



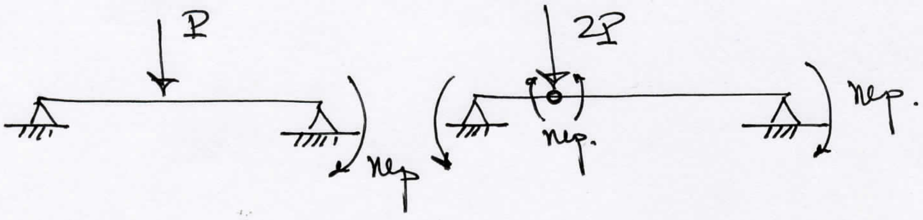
$$M_p = 2\sigma_p \cdot \left[0'25 \cdot 0'3 \cdot \frac{0'25}{2}\right] = 0'01875 \cdot \sigma_p = \boxed{28'25 \text{ KN.m.}}$$

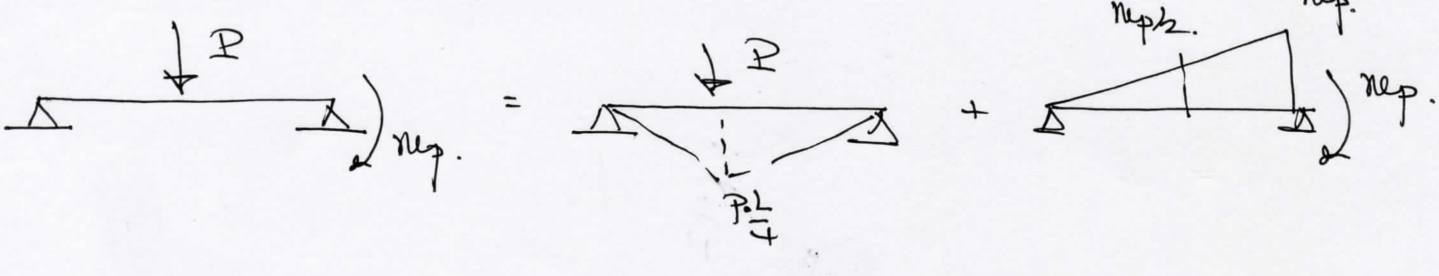
$$P = \frac{3}{4} M_p = \boxed{21'093 \text{ KN.}}$$ carga de colapso.

Mecanismo de colapso:



Ley de flectores en el colapso:

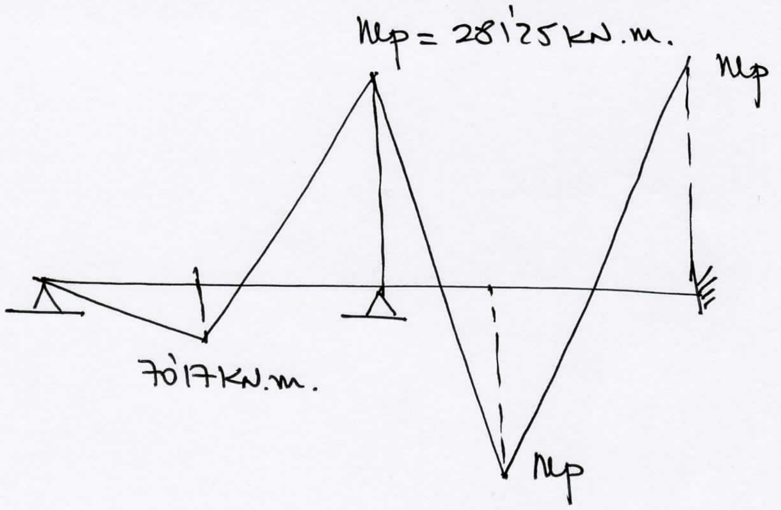




$$\frac{P \cdot L}{4} = \frac{21'093 \cdot 4}{4} = 21'093 \text{ KN.m.}$$

$$\frac{M_p}{2} = \frac{28'125}{2} = 14'076 \text{ KN.m.}$$

$$\left(\frac{P \cdot L}{4} - \frac{M_p}{2}\right) = 21'093 - 14'076 = \boxed{7'017 \text{ KN.m.}}$$


Mp = 28'25 kN.m.
Mp
70'17 kN.m.
Mp

Dada la viga continua de la figura adjunta de rigidez uniforme sometida a un sistema de cargas exteriores en equilibrio se pide hallar:

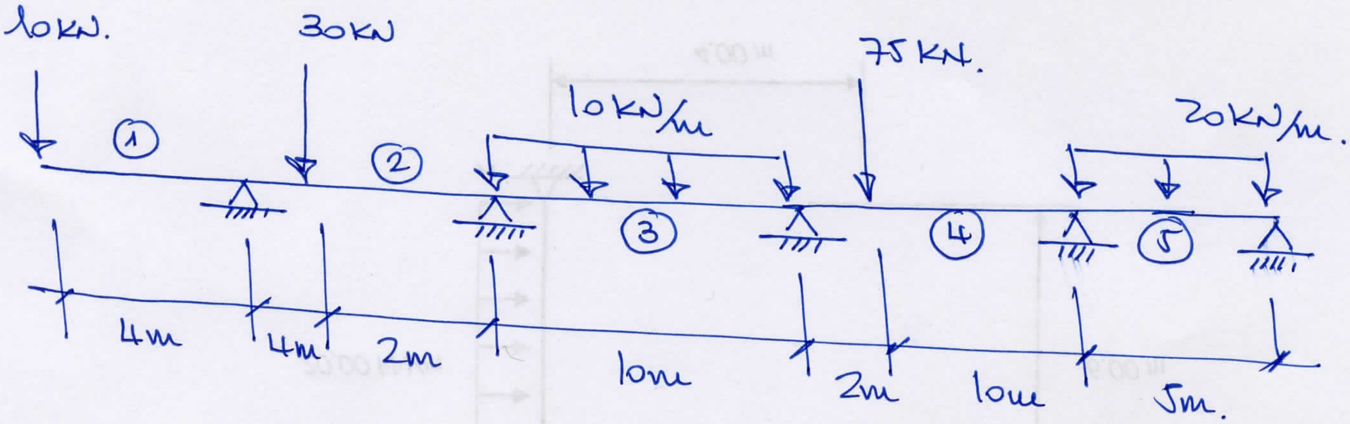
1) El momento plástico y dimensionar la viga sabiendo que su sección es ~~en doble T~~ rectangular de 2a de canto y a de ancho. Sabiendo que ~~[illegible]~~ las cargas dadas producen el agotamiento.

Factor de forma 1'2.

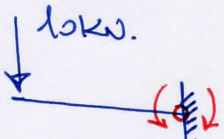
$\sigma_f = 2'4 \cdot 10^5 \; kN/m^2$

2) Mecanismo de colapso.

3) Reacciones y ley de momentos flectores en el instante de colapso.



TRAMO 1.

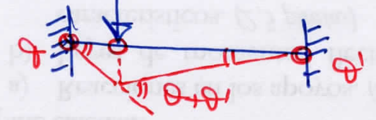


$M_p = P.L = 10.4 = \boxed{40 \; kN.m.}$

TRAMO 2

GH = 2 2

H = 3.



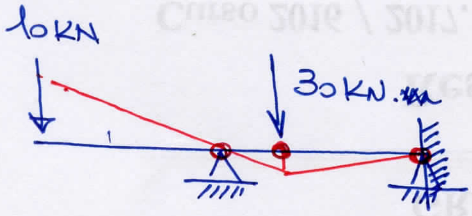
$tg\,\theta = \frac{a}{4}$ $\quad a = 4\theta$ $\quad$ ~~θ'~~ $4\theta = 2\theta'$

$tg\,\theta' = \frac{a}{2}$ $\quad a = 2\theta'$ $\quad \theta' = 2\theta.$

$30.a = M_p\theta + M_p.(\theta+\theta') + M_p.\theta'$

$30.4\theta = M_p.\theta + M_p\, 3\theta + M_p.\,2\theta$

$120 = 6M_p \qquad M_p = \frac{120}{6} = \boxed{20 \; kN.m.}$

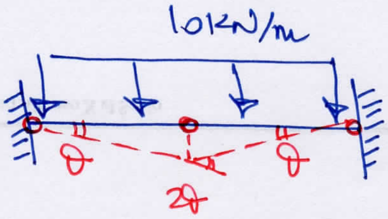


$tg\,\theta = \frac{b}{4} \qquad b = 4\theta.$

$(30.4\theta) - 10.4\theta = M_p.(\theta+\theta') + M_p.\theta'$

$(120\theta - 40\theta) = M_p.\,3\theta + M_p.\,2\theta \qquad 80 = 5M_p \quad \boxed{M_p = 16\,kN}$

TRAMO 3.



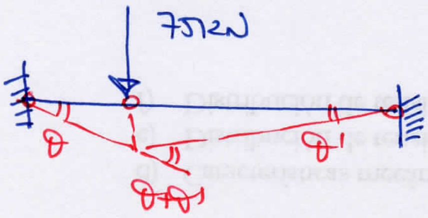
$tg\,\theta = \frac{a}{5}$ $a = 5\theta.$

$AREA = \frac{1}{2} 5\theta \cdot 10 = 25\theta.$

$(10\ \text{kN/m})\ 25\theta = Mp\cdot\theta + Mp\,2\theta + Mp\,\theta$

$250 = 4\,Mp \qquad Mp = \frac{250}{4} = 62'5\ \text{kN·m.}$

TRAMO 4



$tg\,\theta = \frac{a}{2}$ $a = 2\theta$

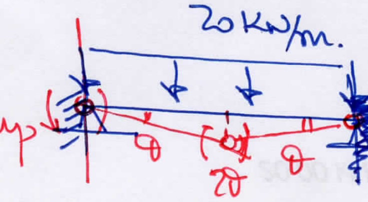
$tg\,\theta' = \frac{a}{10}$ $a = 10\theta'$

$2\theta = 10\theta'$ $\theta' = \frac{1}{5}\theta.$

$(75\cdot 2\theta) = Mp\cdot\theta + Mp(\theta + \frac{1}{5}\theta) + Mp\cdot\frac{1}{5}\theta.$

$750 = 5Mp + 6Mp + Mp \qquad Mp = \frac{750}{12} = 62'5\ \text{kN}$

TRAMO 5.



$tg\,\theta = \frac{a}{2'5}$ $a = 2'5\theta.$

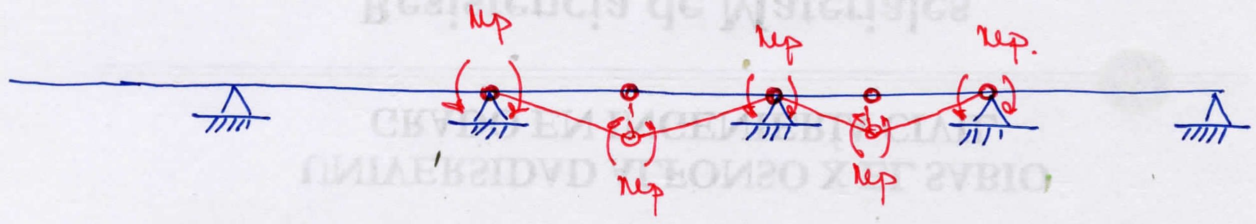
$AREA = \frac{1}{2}\, 2'5\theta \cdot 5 = 6'25\theta$

$(20\cdot 6'25\cdot\theta) = Mp\theta + Mp\,2\theta$ ~~Mpθ~~.

$Mp = 41'6\ \text{kN·m.}$

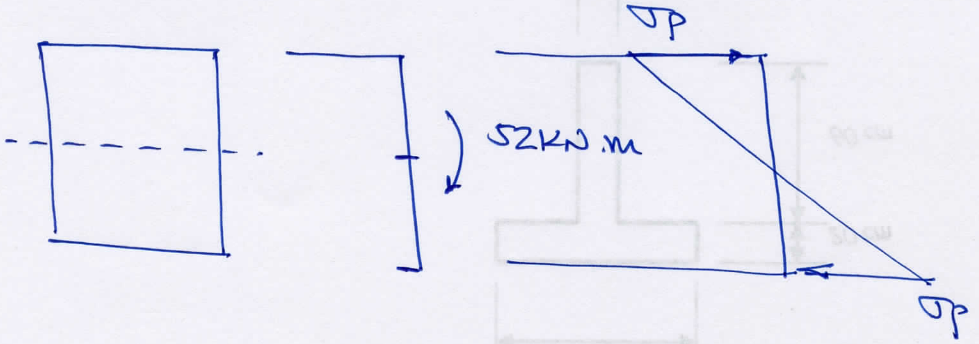
$Mp = 62'5\ \text{kN·m.}$ El momento plástico es el mayor de los necesarios para que la viga resista las cargas en cada tramo.

MECANISMO DE ROTURA:



$M_p = \boxed{625\ kN \cdot m}$

$M_e = \frac{M_p}{\lambda} = \frac{625}{12} = \boxed{52\ kN \cdot m}$



$\boxed{\sigma_p = \frac{M_e \cdot y}{I}}$

LEY FLECTORES.